EXERCISE 1 OPEN PROBLEMS IN NUMBER THEORY 2017/18 DUE DATE: MARCH 21, 2018

Exercise 1. Show that a rational number is not normal (to any base).

Exercise 2. Show that the sequence of fractional parts $\{\{\log n\} : n = n\}$ $1, 2, \dots$ is dense in [0, 1).

Exercise 3. Using summation by parts, show that

$$\sum_{n \le x} \log n = x \log x - x + O(\log x)$$

$$\sum_{n \le x} \log n = x \log x - x + O(\log x)$$

$$\sum_{n=1}^{N} \frac{1}{n} = \log N + C + O(\frac{1}{N})$$

 $for\ some\ constant\ C$.